# Lecture 6: Growth Theory II: The Short and Long Run <br> See Barro Chapter 4 

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## Short Run and Long Run

- We have a framework for thinking about growth

$$
\begin{gathered}
k^{*}=\left(\frac{s \delta+n}{s A}\right)^{\frac{1}{\alpha-1}} \\
\Delta^{*} k=s A k^{\alpha-1}-s \delta-n \\
y=A k^{\alpha} \\
c=s A k^{\alpha}
\end{gathered}
$$

- We want to think about how changes in $k, s, A$, or $n$, will change consumption, production, and capital in the short and long run.


## Shock to Capital

- What happens if, after being at $k^{*}$, we suddenly lose a bunch of capital?

$$
\begin{gathered}
k^{*}=\left(\frac{s \delta+n}{s A}\right)^{\frac{1}{\alpha-1}} \\
\Delta^{*} k=s A k^{\alpha-1}-s \delta-n \\
y=A k^{\alpha} \\
c=s A k^{\alpha}
\end{gathered}
$$

- In the long run, we know nothing has changed because $k^{*}$ hasn't changed.
- In the short run, we can see that when capital goes down
- $y \downarrow$
- $y \downarrow \Rightarrow \Delta^{*} k \uparrow, c \downarrow, r \uparrow$


## Permanent Shock to the Level of Productivity

- Productivity moves from $A$ to $A^{\prime}, A^{\prime}<A$

$$
\begin{gathered}
k^{*}=\left(\frac{s \delta+n}{s A}\right)^{\frac{1}{\alpha-1}} \\
\Delta^{*} k=s A k^{\alpha-1}-s \delta-n \\
y=A k^{\alpha} \\
c=s A k^{\alpha}
\end{gathered}
$$

- In the long run, we now know the level of capital is lower
- We have "too much" capital and "too much" production
- Therefore, we save "too much" and have a lower level of capital tomorrow, higher than long run
- We slowly converge to the steady state from above
- A period of declining consumption, declining capital, high interest rates


## Permanent Shock to Depreciation

- Depreciation moves from $\delta$ to $\delta^{\prime}, \delta^{\prime}<\delta$

$$
\begin{gathered}
k^{*}=\left(\frac{s \delta+n}{s A}\right)^{\frac{1}{\alpha-1}} \\
\Delta^{*} k=s A k^{\alpha-1}-s \delta-n \\
y=A k^{\alpha} \\
c=s A k^{\alpha}
\end{gathered}
$$

- In long run, we now know the level of capital is lower
- We have "too much" capital and "too much" production
- Therefore, we save "too much" and have a lower level of capital tomorrow, higher than long run
- Higher depreciation means we converge more quickly to the steady state
- A period of declining consumption, declining capital, high interest rates


## Permanent Shock to Savings Rate

- Savings moves from $s$ to $s^{\prime}, s^{\prime}>s$

$$
\begin{gathered}
k^{*}=\left(\frac{s \delta+n}{s A}\right)^{\frac{1}{\alpha-1}} \\
\Delta^{*} k=s A k^{\alpha-1}-s \delta-n \\
y=A k^{\alpha} \\
c=s A k^{\alpha}
\end{gathered}
$$

- In long run, we now know the level of capital is higher
- We have "too little" capital
- Our $\Delta^{*} k>0$ compared to what it was
- Slowly converge to new steady state
- At first, lower consumption, then, higher consumption


## Permanent Shock to Labor Growth Rate

- Population growth moves from $n$ to $n^{\prime}, n^{\prime}>n$

$$
\begin{gathered}
k^{*}=\left(\frac{s \delta+n}{s A}\right)^{\frac{1}{\alpha-1}} \\
\Delta^{*} k=s A k^{\alpha-1}-s \delta-n \\
y=A k^{\alpha} \\
c=s A k^{\alpha}
\end{gathered}
$$

- In long run, capital (per worker) is lower
- We have "too much" capital
- Our choice of $\Delta^{*} k$ declines because $n$ increases
- Very slowly converge to new steady state


## Convergence

$>k^{*}=\left(\frac{s \delta+n}{s A}\right)^{\frac{1}{\alpha-1}}$

| Parameter | $k^{*}$ |
| :--- | :--- |
| s | + |
| A | + |
| n | - |
| $\delta$ | - |
| $\mathrm{L}(0)$ | 0 |

## Initial Capital

## Solow Simulation-Levels



## Initial Capital

## Solow Simulation-Pct of US



## Initial Capital

## Growth Rates vs. Initial GDP



## Does Convergence Hold Up?



FIGURE 1.7 Initial income and subsequent growth in Baumol's sample (from DeLong, 1988; used with permission)

Romer figure 1.7
Q: What kind of countries will we have good historical data for?

## Does Convergence Hold Up?



FIGURE 1.8 Initial income and subsequent growth in the expanded sample (from DeLong, 1988; used with permission)

Romer figure 1.8

## Does Convergence Hold Up?



FIGURE 1.9 Initial income and subsequent growth in the postwar period
Romer (3rd ed) figure 1.9

## Does Convergence Hold Up?



FIGURE 1.9 Initial income and subsequent growth in a large sample
Romer (4th ed) figure 1.9

## Does Convergence Hold Up?

Figure 4.10 Growth Rate Versus Level of Real GDP per Person for OECD Countries


Barro figure 4.10.

## Does Convergence Hold Up?

Figure 4.11 Growth Rate Versus Level of Income per Person for U.S.
States, 1880-2000


Barro figure 4.11.

## Why Might Convergence Fail?

- Differences in everything we talked about, except initial capital!


## An Explanation?

Real Per Capita Growth by Country: 1970-198؟


## Sachs Warner 1995

## An Explanation?



Sachs Warner 1995: Let's break things up by "free market"

## An Explanation?

Real Per Capita Growth by Country: 1970-198s


Sachs Warner 1995

## An Explanation?



Sachs Warner 1995

## An Explanation?

- Note that "socialism" isn't your father's socialism
- Typified by state ownership of labor, land, materials
- Typified by state planning and price setting
- What we might call with a broad brush "communism" or "command-and-control"
- Not what we mean by Northern Europe or original EU members


## Conclusion

- We have our first scientific prediction from a model
- Countries growth rates should be linear in log-initial-capital
- Initial success based on bad data
- Failure looking at world at large
- Success looking at advanced/similar countries
- Success looking at states within the US
- Success looking at "free market" economies

